Chapter 4

Techniques Of Circuit Analysis

Mذٍّٛل٠ذ نمٔذٍّٛل٠ذ مذٍّٛب٠ذ
امتحانات سابقة للعديد من المواد آدناه متاحة مجاناً على المواقع المذكورين أدناه

ذٍّٛب سبزٍّٛ٘ل٠ذ الاٌىزشٟٚٔ

ديران هديه عند التنبيه على كل خطا بمذكرات الموقع برسالة SMS أو بالبريد الالكتروني

Materials General:

Physics I/II, Circuits, English 123, Numerical, Dynamics, Strength, Statics

C++, Java, MATLAB, Data Structures, Algorithms, Discrete Math, Digital Logic, Concepts

Mechanical Design I/II, Structural Analysis I/II, Concrete I/II, Soil, Fluid Mechanics, System Dynamics

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اذٔذب سبزٍّٛ٘ل٠ذ الاٌىزشٟٚٔ
In this chapter we introduce these powerful techniques of circuit analysis:

1. Node-voltage method.
3. Source transformation.
4. Thévenin and Norton equivalent circuits.

We consider two other topics that play a role in circuit analysis:

1. Maximum power transfer, considers the conditions necessary to ensure that the power delivered to a resistive load by a source is maximized. Thévenin equivalent circuits are used in establishing the maximum power transfer conditions.

2. Superposition, looks at the analysis of circuits with more than one independent source.

4.1 Terminology:

- A circuit that is drawn with crossing branches is still considered planar if it can be redrawn with no crossover branches.
Example 4.1: Identifying Node, Branch, Mesh and Loop in a Circuit

For the circuit in Fig. 4.3, identify

a) all nodes.
b) all essential nodes.
c) all branches.
d) all essential branches.
e) all meshes.
f) two paths that are not loops or essential branches.
g) two loops that are not meshes.

Solution:

a) The nodes are a, b, c, d, e, f, and g.
b) The essential nodes are b, c, e, and g.
c) The branches are $v_1, v_2, R_1, R_2, R_3, R_4, R_5, R_6, R_7,$ and $I$.
d) The essential branches are $v_1 - R_1, R_2 - R_3, v_2 - R_4, R_5, R_6, R_7,$ and $I$.
e) The meshes are $v_1 - R_1 - R_5 - R_3 - R_2, v_2 - R_2 - R_3 - R_6 - R_4,$ $R_5 - R_7 - R_6,$ and $R_7 - I$.
f) $R_1 - R_5 - R_6$ is a path, but it is not a loop (because it does not have the same starting and ending nodes), nor is it an essential branch (because it does not connect two essential nodes). $v_2 - R_2$ is also a path but is neither a loop nor an essential branch, for the same reasons.
g) $v_1 - R_1 - R_5 - R_6 - R_4 - v_2$ is a loop but is not a mesh, because there are two loops within it. $I - R_5 - R_6$ is also a loop but not a mesh.
Simultaneous Equations:

- The number of unknown currents in a circuit equals the number of branches, \( b \), where the current is not known. For example, the circuit shown in Fig. 4.3 has nine branches in which the currents are unknowns.

- If we let \( n \) represent the number of nodes in the circuit, we can derive \( n - 1 \) independent equations by applying Kirchhoff’s current law to any set of \( n - 1 \) nodes. We must apply Kirchhoff’s voltage law to loops or meshes to obtain the remaining \( b - (n - 1) \) equations.

- These observations also are valid in terms of essential nodes and essential branches. Thus if we let \( n_e \) represent the number of essential nodes and \( b_e \) the number of essential branches where the current is unknown, we can apply Kirchhoff’s current law at \( n_e - 1 \) nodes and Kirchhoff’s voltage law around \( b_e - (n_e - 1) \) loops or meshes.

- As the number of essential nodes is less than or equal to the number of nodes, and the number of essential branches is less than or equal to the number of branches, it is better to use essential nodes and essential branches when analyzing a circuit, because they produce fewer independent equations to solve.

- By introducing new variables, we can describe a circuit with just \( n - 1 \) equations or just \( b - (n - 1) \) equations.

- The new variables are known as node voltages and mesh currents. The node-voltage method enables us to describe a circuit in terms of \( n_e - 1 \) equations; the mesh-current method enables us to describe a circuit \( b_e - (n_e - 1) \) equations.
4.2 **Introduction to the Node-Voltage Method**

1. Make a neat layout of the circuit so that no branches cross over and to mark clearly the essential nodes on the circuit diagram, as in Fig. 4.5. This circuit has three essential nodes \((n_e = 3)\); therefore, we need two \((n_e - 1)\) node-voltage equations to describe the circuit.

2. Select one of the three essential nodes as a reference node and denote it with the symbol ▼. Practically the choice for the reference node often is obvious. For example, the node with the most branches is usually a good choice.

3. Define **node-voltage** on the circuit diagram.

4. Generate the **node-voltage** equations.
   - The node-voltage equation at node 1:
     \[
     \frac{v_1 - 10}{1} + \frac{v_1}{5} + \frac{v_1 - v_2}{2} = 0
     \]
     The node-voltage equation at node 2:
     \[
     \frac{v_2 - v_1}{2} + \frac{v_2}{10} - 2 = 0
     \]

Solving and get: 
\(v_1 = 9.09\) V, \(v_2 = 10.91\) V

Once the node voltages are known, all the branch currents can be calculated. Once these are known, the branch voltages and powers can be calculated.
Example 4.2: Using the Node-Voltage Method

a) Use the node-voltage method of circuit analysis to find the branch currents $i_a$, $i_b$, and $i_c$ in the circuit shown in Fig. 4.8.

b) Find the power associated with each source, and state whether the source is delivering or absorbing power.

Solution:

a) The circuit has two essential nodes

\[
\frac{v_1 - 50}{5} + \frac{v_1}{10} + \frac{v_1}{40} - 3 = 0
\]

\[v_1 = 40 \text{ V}\]

\[i_a = \frac{50 - 40}{5} = 2 \text{ A}\]

\[i_b = \frac{40}{10} = 4 \text{ A}\]

\[i_c = \frac{40}{40} = 1 \text{ A}\]

b) $P_{50V} = -50i_a = -100 \text{ W}$ (delivering)

\[P_{3A} = -3v_1 = -3(40) = -120 \text{ W} \] (delivering)

We check these calculations by noting that total delivered power is 220 W.

The total power absorbed by the three resistors is

\[(i_a)^2(5) + (i_b)^2(10) + (i_c)^2(40) = 220 \text{ W}.

\]
• **Assessment Problem 4.1:**

a) For the circuit shown, use the node-voltage method to find $v_1$, $v_2$, and $i_1$.

b) How much power is delivered to the circuit by the 15 A source?

c) Repeat (b) for the 5 A source.

---

**Solution:**

a) The two node voltage equations are

node 1: $-15 + \frac{v_1 - 0}{60} + \frac{v_1 - 0}{15} + \frac{v_1 - v_2}{5} = 0$

node 2: $5 + \frac{v_2}{2} + \frac{v_2 - v_1}{5} = 0$

Place these equations in standard form:

$v_1 \left(\frac{1}{60} + \frac{1}{15} + \frac{1}{5}\right) + v_2 \left(-\frac{1}{5}\right) = 15$

$v_1 \left(-\frac{1}{5}\right) + v_2 \left(\frac{1}{2} + \frac{1}{5}\right) = -5$

Solving, $v_1 = 60 \text{ V}$ and $v_2 = 10 \text{ V}$

Therefore, $i_1 = \frac{v_1 - v_2}{5} = 10 \text{ A}$

b) $p_{15A} = -(15 \text{ A})v_1 = -(15 \text{ A})(60 \text{ V}) = -900 \text{ W} = 900 \text{ W (delivered)}$

c) $p_{5A} = (5 \text{ A})v_2 = (5 \text{ A})(10 \text{ V}) = 50 \text{ W} = -50 \text{ W (absorbed)}$
4.2 Use the node-voltage method to find \( v \) in the circuit shown.

\[
\begin{align*}
-4.5 + \frac{v_1 - 0}{1} + \frac{v_1 - v_2}{6 + 2} &= 0 \\
\frac{v_2}{12} + \frac{v_2 - v_1}{6 + 2} + \frac{v_2 - 30}{4} &= 0
\end{align*}
\]

Place these equations in standard form:

\[
\begin{align*}
v_1 \left( 1 + \frac{1}{8} \right) + v_2 \left( -\frac{1}{8} \right) &= 4.5 \\
v_1 \left( -\frac{1}{8} \right) + v_2 \left( \frac{1}{12} + \frac{1}{8} + \frac{1}{4} \right) &= 7.5
\end{align*}
\]

Solving, \( v_1 = 6 \) V, \( v_2 = 18 \) V;

To find the voltage \( v \), first find the current \( i \) through the series-connected 6 \( \Omega \) and 2 \( \Omega \) resistors:

\[
i = \frac{v_1 - v_2}{6 + 2} = \frac{6 - 18}{8} = -1.5 \text{ A}
\]

Using the KVL equation, calculate \( v \):

\[
\begin{align*}
-v + 2i + v &= 0 \\
v &= 2i + v_2 = 2(-1.5) + 18 = 15 \text{ V}
\end{align*}
\]
Question 4.2:

4.2 For the circuit shown in Fig. P4.2, state the numerical value of the number of (a) branches, (b) branches where the current is unknown, (c) essential branches, (d) essential branches where the current is unknown, (e) nodes, (f) essential nodes, and (g) meshes.

Solution:

a) 11 branches, 7 with resistors, 2 with independent sources, 2 with dependent sources.
b) The current is unknown in every branch except the one containing the 5 mA current source, so the current is unknown in 10 branches.
c) 11 essential branches each containing a single element.
d) The current is unknown in 10 essential branches.
e) There are 5 nodes - four identified by rectangular boxes and one identified by a triangle.
f) There are 5 essential nodes.
g) A mesh is like a window pane, there are 7 meshes.
Question 4.6:

4.6 Use the node-voltage method to find $v_o$ in the circuit in Fig. P4.6.

Figure P4.6

\[
\frac{v_o - 60}{10} + \frac{v_o}{5} + 3 = 0
\]

\[v_o = 10 \text{ V}\]

Solution:

Applying node-voltage equation derived at node 1:

\[
\frac{v_o - 60}{10} + \frac{v_o}{5} + 3 = 0
\]

\[v_o = 10 \text{ V}\]
Question 4.9:

4.9 Use the node-voltage method to find $v_1$ and $v_2$ in the circuit shown in Fig. P4.9.

![Figure P4.9](image)

Solution:

\[
2.4 + \frac{v_1}{125} + \frac{v_1 - v_2}{25} = 0
\]
\[
v_2 - \frac{v_1}{25} + \frac{v_2}{250} + \frac{v_2}{375} = 3.2 \quad 0
\]

Solving, $v_1 = 25$ V; $v_2 = 90$ V

Check

\[
p_{125\Omega} = \frac{(V)^2}{R_{125\Omega}} = 5 \quad W
\]
\[
p_{25\Omega} = \frac{(90 - 25)^2}{25} = 169 \quad W
\]
\[
p_{250\Omega} = \frac{(90)^2}{250} = 32.4 \quad W
\]
\[
p_{375\Omega} = \frac{(90)^2}{375} = 21.6 \quad W
\]
\[
p_{2.4A} = (V_1)(2.4) = 60 \quad W
\]
\[
\sum p_{\text{abs}} = 5 + 169 + 32.4 + 21.6 + 60 = 288 \quad W
\]
\[
\sum p_{\text{dev to load}} = (90)(3.2) = 288 \quad W
\]

(CHECKS)

Remember:

\[
P = vi
\]
\[
= i^2R
\]
\[
= \frac{V^2}{R}
\]
**Question 4.10:**

a) Use the node-voltage method to find the branch currents $i_a, i_c$ in the circuit shown in Fig. P4.10.

b) Find the total power developed in the circuit.

**Solution:**

a)

\[
\begin{align*}
\frac{v_1 - 128}{8} + \frac{v_1}{48} + \frac{v_1 - v_2}{18} &= 0 \\
\frac{v_2 - v_1}{18} + \frac{v_2}{20} + \frac{v_2 - 70}{10} &= 0
\end{align*}
\]

In standard form,

\[
\begin{align*}
v_1 \left( \frac{1}{8} + \frac{1}{48} + \frac{1}{18} \right) + v_2 \left( -\frac{1}{18} \right) &= \frac{128}{8} \\
v_1 \left( -\frac{1}{8} \right) + v_2 \left( \frac{1}{18} + \frac{1}{20} + \frac{1}{10} \right) &= \frac{70}{10}
\end{align*}
\]

Solving, $v_1 = 96$ V; $v_2 = 60$ V

\[
\begin{align*}
i_a &= \frac{128 - 96}{8} = 4 \text{ A} \\
i_b &= \frac{96}{48} = 2 \text{ A} \\
i_c &= \frac{96 - 60}{18} = 2 \text{ A} \\
i_d &= \frac{60}{20} = 3 \text{ A} \\
i_e &= \frac{60 - 70}{10} = -1 \text{ A}
\end{align*}
\]

b) $p_{\text{dev}} = 128(4) + 70(1) = 582$ W
### 4.3 The Node-Voltage Method and Dependent Sources

- We can handle this state as usual, nothing is new.

#### Example 4.3: Using the Node-Voltage Method with Dependent Sources

Use the node-voltage method to find the power dissipated in the $5 \, \Omega$ resistor in the circuit shown in Fig. 4.10.

![Figure 4.10](image)

**Solution:**
- The circuit has three essential nodes.
- Four branches terminate on the lower node, so we select it as the reference node.

\[
\begin{align*}
\frac{v_1 - 20}{2} + \frac{v_1}{20} + \frac{v_1 - v_2}{5} &= 0 \\
\frac{v_2 - v_1}{5} + \frac{v_2}{10} + \frac{v_2 - 8i_\phi}{2} &= 0 \\
i_\phi &= \frac{v_1 - v_2}{5}
\end{align*}
\]

**Solving:**
- $v_1 = 16 \, V$
- $v_2 = 10 \, V$
- $i_\phi = \frac{16 - 10}{5} = 1.2 \, A$
- $P_{5\Omega} = (1.2)^2(5) = 7.2 \, W$
• **Assessment 4.3:** Understand and be able to use the node-voltage method

a) Use the node-voltage method to find the power associated with each source in the circuit shown.

b) State whether the source is delivering power to the circuit or extracting power from the circuit.

---

**Solution:**

a) The node voltage equations are:

\[
\frac{v_1 - 50}{6} + \frac{v_1}{8} + \frac{v_1 - v_2}{2} - 3i_1 = 0
\]

\[-\frac{5}{4} + \frac{v_2}{4} + \frac{v_2 - v_1}{2} + 3i_1 = 0
\]

Solving with:

\[i_1 = \frac{50 - v_1}{6}\]

\[v_1 = 32 \text{ V}; \quad v_2 = 16 \text{ V}; \quad i_1 = 3 \text{ A}\]

\[p_{50V} = -50i_1 = -150 \text{ W} \quad \text{(Delivering)}\]

\[p_{5A} = -5(v_2) = -80 \text{ W} \quad \text{(Delivering)}\]

\[p_{3i_1} = 3i_1(v_2 - v_1) = -144 \text{ W} \quad \text{(Delivering)}\]

b) all three resources are delivering power to the circuit.
4.4 The Node-Voltage Method Some Special Cases

- When a voltage source is the only element between two essential nodes, the node-voltage method is simplified. In Fig. (4.12) there are three essential nodes, which means that two simultaneous equations are needed. But the 100 V source constrains the voltage between node 1 and the reference node to 100 V. This means that there is only one unknown node voltage ($v_2$). Solution of this circuit thus involves only a single node-voltage equation at node 2:

$$\frac{v_2 - v_1}{10} + \frac{v_2}{50} - 5 = 0, \quad \text{Where} \quad v_1 = 100 \text{ V}$$

$$v_2 = 125 \text{ V}$$

Knowing $v_2$, we can calculate the current in every branch.

- In general, when you use the node-voltage method to solve circuits that have voltage sources connected directly between essential nodes, the number of unknown node voltages is reduced.
For Fig. 4.13:

1. \( n_e = 4 \), So we need \( 4 - 1 = 3 \) essential equations.

2. However, 2 essential nodes are connected by an independent voltage source, which holds our special case.

3. So, the 2 other essential nodes which are connected by CCVS (current control voltage source) illustrate that we only need 1 unknown node voltage equation.

At node 2:

\[
\frac{v_2 - v_1}{5} + \frac{v_2}{50} + i = 0
\]

at node 1:

\( v_1 = 50 \text{ V} \)

at node 3:

\[
\frac{v_3}{100} - i - 4 = 0
\]

We eliminate \( i \) simply by adding Eqs.

\[
\frac{v_2 - v_1}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0
\]
The Concept of a Supernode:

- Last equation may be written directly. We consider nodes 2 and 3 to be a single node and simply sum the currents away from the node in terms of the node voltages $v_2$ and $v_3$. Figure 4.15 illustrates this approach.

- When a voltage source is between two essential nodes, we can combine those nodes to form a supernode.

- Combining node 2, 3 to form a supernode, we can write:

\[
\frac{v_2 - v_1}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0
\]

Which is same as (1)

- We can eliminate $v_1$ from the equation because we know that $v_1 = 50$ V. Next express $v_3$ as a function of $v_2$:

$$v_3 = v_2 + 10i_\phi$$

also

$$i_\phi = \frac{v_2 - 50}{5}$$

Solving,

$$v_2 = 60 \text{ V}$$

$$i_\phi = \frac{60 - 50}{5} = 2 \text{ A}$$

$$v_3 = 60 + 20 = 80 \text{ V}$$
Assessment 4.4:

Use the node-voltage method to find $V_o$ in the circuit shown.

\[
\frac{V_o}{40} + \frac{V_o - 10}{10} + \frac{V_o - (-20i_\Delta)}{20} = 0
\]

Applying KCL at node a:

\[-i_\Delta + i_{10\Omega} + i_{30\Omega} = 0
\]

\[i_\Delta = \frac{10 - V_o}{10} + \frac{10 + 20i_\Delta}{30}
\]

Solving the two equations gives:

\[i_\Delta = -3.2 \text{ A}; \quad V_o = 24 \text{ V}
\]

Thus, the dependent source is delivering 36 W, or absorbing -36 W.
Assessment 4.5:

Use the node-voltage method to find \( v \) in the circuit shown.

\[
\frac{v_1}{7.5} + \frac{v_1 - v}{2.5} - 4.8 = 0
\]
\[
v - v_1 + \frac{v}{10} + \frac{v_2}{2.5} + \frac{v_2 - 12}{1} = 0
\]

Solving with:

\[
i_x = \frac{v_1}{7.5}
\]
\[
v + i_x = v_2
\]

\( v_1 = 15 \text{ V}, \quad v_2 = 10 \text{ V}, \quad i_x = 2 \text{ A}, \quad v = 8 \text{ V} \)
Assessment 4.6:
Use the node-voltage method to find $v_1$ in the circuit shown.

The node voltage equation at $v_1$ is:

$$\frac{v_1 - 60}{2} + \frac{v_1}{24} + \frac{v_1 - (60 + 6i_\phi)}{3} = 0$$

The constrain equation due to the dependent source is:

$$i_\phi = \frac{(60 + 6i_\phi) - v_1}{3}$$

Solving,

$$i_\phi = -4 \text{ A} \quad \text{and} \quad v_1 = 48 \text{ V}$$
Question 4.25:

Use the node-voltage method to find the value of \( v_o \) in the circuit in Fig. P4.25.

Solution:

Before writing Node-Voltage equations, we can conclude:

\[
\begin{align*}
    v_1 - 24 &= 0 \\
    v_1 &= 24 \text{ V} \\
    v_2 &= v_\Delta \\
\end{align*}
\]

Writing Node-Voltage equations:

\[
\begin{align*}
    -10 + \frac{v_2}{6} + \frac{v_2 - v_3}{2} &= 0 \\
    \frac{v_3 - v_2}{2} + \frac{v_3 - 24}{4} + \frac{2v_2}{3} &= 0 \\
\end{align*}
\]

Solving,

\[
\begin{align*}
    v_2 &= 18 \text{ V; } \\
    v_3 &= 4 \text{ V} \\
    v_1 - v_o &= v_3 \\
    v_o &= 24 - v_3 = 20 \text{ V} \\
\end{align*}
\]
4.5 **Introduction to the Mesh-Current Method:**

- **Mesh:** a loop with no other loops inside it.
- This method can be used only for planar circuits.
- Can describe the circuit in terms of $b_e - (n_e - 1)$ equations where $(b_e \equiv \textit{number of essential nodes})$.
  $(n_e \equiv \textit{number of essential meshes})$.
- The circuit in Fig. 4.18 contains seven essential branches where the current is unknown, and four essential nodes.
  We must write four $[7 - (4 - 1)]$ mesh-current equations.
- **A mesh current:** is a current that exists only in the perimeter of a mesh.

- **Writing circuit equations using Mesh-Current Method for a simple circuit:**

  \[-v_1 + R_1 i_a + R_3 (i_a - i_b) = 0\]
  \[v_2 + R_3 (i_b - i_a) + R_2 i_b = 0\]

  ![Diagram of mesh-current method](image)

  **Figure 4.18:** The circuit shown in Fig. 4.1(b), with the mesh currents defined.

  **Figure 4.20:** Mesh currents $i_a$ and $i_b$.
Solution:

a) Figure 4.22 shows the three mesh currents used to describe the circuit.

\[-40 + 2i_a + 8(i_a - i_b) = 0\]
\[8(i_b - i_a) + 6i_b + 6(i_b - i_c) = 0\]
\[6(i_c - i_b) + 4i_c + 20 = 0\]

Solving,

\[i_a = 5.6 \text{ A}\]
\[i_b = 2.0 \text{ A}\]
\[i_c = -0.80 \text{ A}\]

\[P_{40V} = -40i_a = -224 \text{ W} \quad \text{(Deliver)}\]
\[P_{20V} = 20i_c = -16 \text{ W} \quad \text{(Deliver)}\]

b) The branch current in the 8 Ω resistor in the direction of the voltage drop

\[v_o = i_a - i_b \quad \text{Therefore}\]
\[v_o = 8(i_a - i_b) = 8(3.6) = 28.8 \text{ V}\]
Assessment Problem 4.7: Understand and be able to use mesh-current method

Use the mesh-current method to find (a) the power delivered by the 80 V source to the circuit shown and (b) the power dissipated in the 8 Ω resistor.

Solution:

The mesh current equations are:

\[-80 + 5(i_1 - i_2) + 26(i_1 - i_3) = 0\]
\[30i_2 + 90(i_2 - i_3) + 5(i_2 - i_1) = 0\]
\[8i_3 + 26(i_3 - i_1) + 90(i_3 - i_2) = 0\]

Solving,

\[i_1 = 5 \text{ A}; \quad i_2 = 2 \text{ A}; \quad i_3 = 2.5 \text{ A}\]

a) \[P_{80V} = -80 \times (5) = -400 \text{ W} \quad \text{(deliver)}\]

b) \[P_{8\Omega} = (8) \times (2.5)^2 = 50 \text{ W}\]
Question 4.31:

a) Use the mesh-current method to find the branch currents $i_a$, $i_b$, and $i_c$ in the circuit in Fig. P4.31. 

b) Repeat (a) if the polarity of the 64 V source is reversed.

Solution:

a) We need 2 mesh equations:

\[-40 + 3i_1 + 45(i_1 - i_2) + 2i_1 = 0\]
\[-64 + 1.5i_2 + 45(i_2 - i_1) + 4i_2 = 0\]

Solving,

\[50i_1 - 45i_2 = 40 \quad \rightarrow (1)\]
\[-45i_1 + 50.5i_2 = 64 \quad \rightarrow (2)\]

\[i_1 = 9.8 \text{ A,} \quad i_2 = 10 \text{ A}\]
\[i_a = i_1 = 9.8 \text{ A,} \quad i_c = -i_2 = -10 \text{ A}\]
\[i_b = i_a - i_c = 9.8 - 10 = -0.2 \text{ A}\]

b) Change (64) to (−64) in equation (2)

Solving,

\[i_1 = -1.72 \text{ A,} \quad i_2 = -2.8 \text{ A}\]
\[i_a = i_1 = -1.72 \text{ A,} \quad i_c = -i_2 = 2.8 \text{ A}\]
\[i_b = i_1 - i_c = -1.72 - 2.8 = 1.08 \text{ A}\]
**Question 4.32:**

a) Use the mesh-current method to find the total power developed in the circuit in Fig. P4.32.

b) Check your answer by showing that the total power developed equals the total power dissipated.

**Solution:**

a) We need 3 mesh equations:

\[-110 + 10(i_1 - i_2) + 3(i_1 - i_3) - 12 + 4i_1 = 0\]
\[6i_2 + 12(i_2 - i_3) + 10(i_2 - i_1) = 0\]
\[70 + 2i_3 + 12 + 3(i_3 - i_1) + 12(i_3 - i_2) = 0\]

Solving,
\[17i_1 - 10i_2 - 3i_3 = 122 \rightarrow (1)\]
\[-10i_1 + 28i_2 - 12i_3 = 0 \rightarrow (2)\]
\[-3i_1 - 12i_2 + 17i_3 = -82 \rightarrow (3)\]

\[i_1 = 8 \text{ A}, \quad i_2 = 2 \text{ A}, \quad i_3 = -2 \text{ A}\]

\[P_{110V} = -110 \times 8 = -880 \text{ W} \quad \text{(deliver)}\]
\[P_{12V} = -12 \times (8 - (-2)) = -120 \text{ W} \quad \text{(deliver)}\]
\[P_{70V} = 70 \times (-2) = -140 \text{ W} \quad \text{(deliver)}\]

\[\sum P_{deliver} = 1140 \text{ W}\]

b) \[P_{4\Omega} = (8)^2(4) = 256 \text{ W}\]
\[P_{10\Omega} = (6)^2(10) = 360 \text{ W}\]
\[P_{12\Omega} = (-4)^2(12) = 192 \text{ W}\]
\[P_{2\Omega} = (-2)^2(2) = 8 \text{ W}\]
\[P_{6\Omega} = (2)^2(6) = 24 \text{ W}\]
\[P_{3\Omega} = (10)^2(3) = 300 \text{ W}\]

\[\sum P_{abs} = 1140 \text{ W}\]
4.6 The Mesh-Current Method and Dependent Sources

- If the circuit contains dependent sources, the mesh-current equations must take in consideration the appropriate equations.

**Example 4.5:** Using the Mesh-Current Method with Dependent Sources

![Circuit Diagram]

Use the mesh-current method of circuit analysis to determine the power dissipated in the 4 Ω resistor in the circuit shown in Fig. 4.23.

**Solution:**

We need three mesh current equations to describe the circuit.

\[-50 + 5(i_1 - i_2) + 20(i_1 - i_3) = 0\]
\[5(i_2 - i_1) + 1i_2 + 4(i_2 - i_3) = 0\]
\[20(i_3 - i_1) + 4(i_3 - i_2) + 15i_φ = 0\]

Solving with:

\[i_φ = i_1 - i_3\]
\[\therefore i_2 = 26 \text{ A}; \quad i_3 = 28 \text{ A}\]

\[p_{4Ω} = (i_3 - i_2)^2(4) = (2)^2(4) = 16 \text{ W}\]
• **Assessment 4.8:** Understand and be able to use the mesh-current method

a) Determine the number of mesh-current equations needed to solve the circuit shown.

b) Use the mesh-current method to find how much power is being delivered to the dependent voltage source.

---

**Solution:**

\[
\begin{align*}
    b &= 6, & n &= 4, & b - (n - 1) = 6 - (4 - 1) = 6 - 3 = 3 \\
    & \quad -25 + 2(i_1 - i_2) + 5(i_1 - i_3) + 10 = 0 \\
    & \quad -3v_\theta + 14i_2 + 3(i_2 - i_3) + 2(i_2 - i_1) = 0 \\
    & \quad 1i_3 - 10 + 5(i_3 - i_1) + 3(i_3 - i_2) = 0
\end{align*}
\]

Solving with:

\[v_\theta = 3(i_3 - i_2)\]

\[i_1 = 4 \ A; \quad i_2 = -1 \ A; \quad i_3 = 3 \ A; \quad v_\theta = 12 \ V\]

\[p_{ds} = -(-3v_\theta)i_2 = 3(12)(-1) = -36 \ W \ (deliver)\]
Assessment 4.9: Understand and be able to use the mesh-current method

Use the mesh-current method to find $v_o$ in the circuit shown.

\[
-25 + 6(i_a - i_b) + 8(i_a - i_c) = 0
\]
\[
2i_b + 8(i_b - i_c) + 6(i_b - i_a) = 0
\]
\[
5i_\theta + 8(i_c - i_a) + 8(i_c - i_b) = 0
\]

Solving with:

\[
i_\theta = i_a
\]
\[
i_a = 4 \text{ A}; \quad i_b = 2.5 \text{ A}; \quad i_c = 2 \text{ A}
\]
\[
v_o = 8(i_a - i_c) = 8(4 - 2) = 16 \text{ V}
\]
Question 4.37:

Use the mesh-current method to find the power dissipated in the 8 Ω resistor in the circuit in Fig. P4.37.

Figure P4.37

Solution:

\[-80 + 16(i_1 - i_2) - 7(i_1 - i_3) + 8i_1 = 0\]
\[7i_2 + 4(i_2 - i_3) + 16(i_2 - i_1) = 0\]
\[4(i_3 - i_2) + 24i_\sigma + 20i_3 + 7(i_3 - i_1) = 0\]

Solving with:

\[i_\sigma = i_2\]
\[i_1 = 3.5 \text{ A}\]

\[p_{8\Omega} = (3.5)^2(8) = 98 \text{ W}\]
• **Question 4.38:**

Use the mesh-current method to find the power delivered by the dependent voltage source in the circuit seen in Fig. P4.38.

Figure P4.38

![Image of a circuit diagram](image)

• **Solution:**

\[-660 + 5i_1 + 15(i_1 - i_3) + 10(i_1 - i_2) = 0\]
\[-20i_\Phi + 10(i_2 - i_1) + 50(i_2 - i_3) = 0\]
\[25i_3 + 50(i_3 - i_2) + 15(i_3 - i_1) = 0\]

Solving with:

\[i_\Phi = i_2 - i_3\]

\[\therefore i_1 = 42 \text{ A}; \quad i_2 = 27 \text{ A}; \quad i_3 = 22 \text{ A}; \quad i_\Phi = 5 \text{ A}\]

\[p_{20i_\Phi} = vi_2 = 20i_\Phi i_2 = 20(5)(27) = 2700 \text{ W} \quad \text{(developed)}\]

\[p_{660V} = vi_1 = 660(42) = 27,720 \text{ W} \quad \text{(developed)}\]

Check:

\[\sum p_{dev} = 27,720 + 2700 = 30,420 \text{ W}\]

\[\sum p_{dis} = (42)^2(5) + (22)^2(25) + (20)^2(15) + (5)^2(50) + (15)^2(10)\]

\[= 30,420 \text{ W}\]
**Question 4.39:**

Use the mesh-current method to find the power developed in the dependent voltage source in the circuit in Fig. P4.39.

![Circuit Diagram](image)

**Solution:**

\[
2.65v_\Delta + 25(i_1 - i_3) + 15(i_1 - i_2) = 0 \\
15(i_2 - i_1) + 100(i_2 - i_3) + 35i_2 + 125 = 0 \\
-125 + 85i_3 + 100(i_3 - i_2) + 25(i_3 - i_1) = 0
\]

Solving with:

\[
v_\Delta = 100(i_2 - i_3) \\
i_1 = 7 \text{ A;} \quad i_2 = 1.2 \text{ A;} \quad i_3 = 2 \text{ A} \\
v_\Delta = 100(1.2 - 2) = -80 \text{ V} \\
P_{2.65v_\Delta} = (2.65 \times -80) \times 7 = -1,485 \text{ W } (\text{delivers})
\]
4.7 Introduction to SuperMesh Method:

- We can use the concept of **supermesh** if there exist a current source between 2 loops.
- If we solve the circuit in fig. 4.25 using previous **Mesh-Current Method** the first two equations are:

For mesh a: 
\[-100 + 3(i_a - i_b) + v + 6i_a = 0\]
For mesh c: 
\[50 + 4i_c - v + 2(i_c - i_b) = 0\]

Adding them to eliminate \(v\) we get:
\[-50 + 9i_a - 5i_b + 6i_c = 0\]

**The Concept of a Supermesh Method:**

- To create a supermesh, we mentally remove the current source from the circuit by simply avoiding this branch when writing the mesh-current equation.

- If we solve the circuit in fig. 4.25 using **Super-Mesh Method** we can get the last equation directly:

\[-100 + 3(i_a - i_b) + 2(i_c - i_b) + 50 + 4i_c + 6i_a = 0\]
\[\quad - 50 + 9i_a - 5i_b + 6i_c = 0\]
**Question 4.41:**

a) Use the mesh-current method to find how much power the 12 A current source delivers to the circuit in Fig. P4.41.
b) Find the total power delivered to the circuit.
c) Check your calculations by showing that the total power developed in the circuit equals the total power dissipated.

**Solution:**

\[
\begin{align*}
-600 + 10i_1 + 40(i_1 - i_2) + 14(i_1 - i_3) &= 0 \\
400 + 2(i_2 - i_3) + 40(i_2 - i_1) + 8i_2 &= 0 \\
i_3 &= -12
\end{align*}
\]

Solving: \(i_1 = 2.9\) A; \(i_2 = -6.16\) A

a) \(v_{12A} + 14(i_3 - i_1) + 2(i_3 - i_2) = -220.28\) V

\(v_{12A} = 220.28\) V, \(p_{12A} = vi_3 = 220.28(-12) = -2,643.36\) W (deliver)

b) \(p_{400V} = 400i_2 = 400(-6.16) = -2,643.36\) W (deliver)

\(p_{600V} = -60i_1 = -6(2.9) = -1740\) W (deliver)

\[\sum p_{\text{developed}} = -6847.36\) W (deliver)

C) \(\sum p_{\text{dissipated}} = \sum i^2R\)

\[= (2.9)^2(10) + (6.16)^2(8) + (9.06)^2(40) + (14.9)^2(14) + (5.84)^2(2)\]

\[= 6847.36\) W

\[\sum p_{\text{dissipated}} = 6847.36\) W = \(\sum p_{\text{developed}}\) (CHECKS)
- **Question 4.43:**

Use the mesh-current method to find the total power developed in the circuit in Fig. P4.43.

- **Solution:**

Mesh equations:

\[ 7i_1 + 1(i_1 - i_3) + 2(i_1 - i_2) = 0 \]
\[ -90 + 2(i_2 - i_1) + 3(i_2 - i_3) + 165 = 0 \]

Constrain equations:

\[ i_3 = 0.5v_\Delta; \quad v_\Delta = 2(i_2 - i_1) \]

Solving,

\[ i_1 = -9 \text{ A}; \quad i_2 = -33 \text{ A}; \quad i_3 = -24 \text{ A}; \quad v_\Delta = -48 \text{ V} \]

Using KVL to calculate \( v_{CS} \):

\[ v_{CS} - 165 + 3(i_3 - i_2) + 1(i_3 - i_1) = 0 \]

\[ v_{CS} = 153 \text{ V} \]

\[ p_{90V} = -(90)(-33) = 2970 \text{ W} \]

\[ p_{165V} = (165)(i_3 - i_2) = -1485 \text{ W} \]

\[ p_{dep \ source} = (153)[0.5(v_\Delta)] = -3672 \text{ W} \]

Thus, the total power developed is: \( 1485 + 3672 = 5157 \text{ W} \)
• **Question 4.44:**

Use the mesh-current method to find the total power developed in the circuit in Fig. P4.44.

![Figure P4.44](image)

\[ 5i_1 + 20(i_1 - i_2) + 25(i_1 - i_g) = 0 \]
\[ 20(i_2 - i_1) - 30i_\Delta + 100(i_2 - i_g) = 0 \]

**Constraint equations:**

\[ i_g = 4 \text{ A}; \quad i_\Delta = i_1 \]

**Solving,**

\[ i_1 = 4 \text{ A}; \quad i_2 = 5 \text{ A} \]
\[ -v_{4A} + 25(i_g - i_1) + 100(i_g - i_2) = 0 \]
\[ v_{4A} = 25(0) + 100(-1) = -100 \text{ V} \]
\[ p_{4A} = -vi = -(-100)(4) = 400 \text{ W} \quad (\text{delivers}) \]
\[ = 400 \text{ W} \quad (\text{absorbs}) \]
\[ p_{-30i_\Delta} = -vi = -(30 \times 4)(5) = -600 \text{ W} \quad (\text{delivers}) \]
\[ \sum p_{\text{developed}} = -600 + 400 = -200 \text{ W} \quad (\text{delivered}) \]
Question 4.51:

a) Find the branch currents $i_a - i_c$ for the circuit shown in Fig. P4.51.

b) Check your answers by showing that the total power generated equals the total power dissipated.

Solution:

\[ 40(i_3 - i_1) + 10(i_3 - i_2) + 35(i_4 - i_2) + 150 = 0 \]
\[ 35(i_2 - i_4) + 10(i_2 - i_3) + 15i_d = 0 \]

\[ 3i_a = i_3 - i_4; \quad i_a = i_1 - i_3 \]
\[ i_d = i_4; \quad i_1 = 30 \text{ A} \]

Solving, $i_1 = 30 \text{ A}; \quad i_2 = 8 \text{ A}; \quad i_3 = 24 \text{ A}; \quad i_4 = 6 \text{ A}$

\[ i_a = 30 - 24 = 6 \text{ A}; \quad i_b = 8 - 24 = -16 \text{ A}; \quad i_c = 8 - 6 = 2 \text{ A}; \quad i_d = 6 \text{ A}; \quad i_e = i_2 = 8 \text{ A} \]
Continued (Question 4.51):

b) \( v_a = 40i_a = 240 \text{ V}; \quad v_b = 150 - 35i_c = 80 \text{ V} \)

\[
p_{30\Omega} = -30v_a = -30(240) = -7200 \text{ W} \quad \text{(generated)}
\]

\[
p_{15i_d} = 15i_di_e = 15(6)(8) = 720 \text{ W} \quad \text{(dissipated)}
\]

\[
p_{3i_a} = 3i_a v_b = 3(6)(80) = 1440 \text{ W} \quad \text{(dissipated)}
\]

\[
p_{150\Omega} = 150i_d = 150(6) = 900 \text{ W} \quad \text{(dissipated)}
\]

\[
p_{40\Omega} = (6)^2(40) = 1440 \text{ W} \quad \text{(dissipated)}
\]

\[
p_{10\Omega} = (-16)^2(10) = 2560 \text{ W} \quad \text{(dissipated)}
\]

\[
p_{35\Omega} = (2)^2(35) = 140 \text{ W} \quad \text{(dissipated)}
\]

\[
\sum P_{\text{delivered}} = 7200 \text{ W}
\]

\[
\sum P_{\text{dissipated}} = 720 + 1440 + 900 + 1440 + 2560 + 140 = 7200 \text{ W}
\]
4.9 Source Transformations:

- A source transformation is a method that can be used to simplify circuits.

- A source transformation, shown in Fig. 4.36, allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor or vice versa.

- The relationship between $v_s$ and $i_s$ that guarantees the two configurations in Fig. 4.36 to be equivalent with respect to nodes a, b.

\[
i_s = \frac{v_s}{R}
\]

- If the polarity of $v_s$ is reversed, the orientation of $i_s$ must be reversed to maintain equivalence.

- What happens if there is a resistance $R_p$ in parallel with the voltage source or a resistance $R_s$, in series with the current source?

In both cases, the resistance has no effect on the equivalent circuit that predicts behavior with respect to terminals a, b.

Figure 4.39 summarizes this observation.
**Example 4.8: Using Source Transformation to Solve a Circuit**

a) For the circuit shown in Fig. 4.37, find the power associated with the 6 V source.

b) State whether the 6 V source is absorbing or delivering the power calculated in (a).

**Solution:**

- We must reduce the circuit in a way that preserves the identity of the branch containing the 6 V source. We can transform the 40 V source in series with the 5 Ω resistor into an 8 A current source in parallel with a 5 Ω resistor, as shown in Fig. (a).

- Next, we can replace the parallel combination of the 20 Ω and 5 Ω resistors with a 4 Ω resistor. This 4 Ω resistor is in parallel with the 8 A source and therefore can be replaced with a 32 V source in series with a 4 Ω resistor, as shown in Fig. (b).

- We can continue in the same manner as in Figures (c) & (d).

\[
i = \frac{19.2 - 6}{16} = 0.825 \text{ A}
\]

\[
p_{6V} = (0.825)(6) = 4.95 \text{ W (absorb)}
\]
Example 4.9: Using Special Source Transformation Techniques

a) Use source transformations to find the voltage \( v_o \) in the circuit shown in Fig. 4.40.

b) Find the power developed by the 250 V voltage source.

c) Find the power developed by the 8 A current source.

---

Solution: a)

- We begin by removing the 125 Ω and 10 Ω resistors, because the 125 Ω resistor is connected across the 250 V voltage source and the 10 Ω resistor is connected in series with the 8 A current source.

- We also combine the series-connected resistors into a single resistance of 20 Ω.

- We can use a source transformation to replace the 250 V source and 25 Ω resistor with a 10 A source in parallel with the 25 Ω resistor.

- We can now simplify the circuit shown in Fig. 4.42 by using Kirchhoff’s current law to combine the parallel current sources into a single source. The parallel resistors combine into a single resistor. Figure 4.43 shows the result.

Hence: \( v_o = 20 \) V
b) The current supplied by the 250 V source equals the current in the 125 Ω resistor plus the current in the 25 Ω resistor. Thus:

\[ i_s = \frac{250}{125} = \frac{250 - 20}{25} = 11.2 \text{ A} \]

\[ P_{250\text{V}(\text{developed})} = (250)(11.2) = 2800 \text{ W} \]

c) To find the power developed by the 8 A current source, we first find the voltage across the source. Let \( v_s \) represents the voltage across the source, positive at the upper terminal:

\[ v_s + 8(10) = v_o = 20, \quad \text{or} \quad v_s = -60 \text{ V} \]

\[ \therefore P_{8\text{A}(\text{developed})} = (-60)(8) = -480 \text{ W} \]

Note that: The 125 Ω and 10 Ω resistors do not affect the value of \( v_o \) but do affect the power calculations.
Assessment 4.15: Understand Source Transformation

a) Use a series of source transformations to find the voltage $v$ in the circuit shown.

b) How much power does the 120 V source deliver to the circuit?

Solution: a)

- Transform the 120 V source in series with the 20 $\Omega$ resistor into a 6 A source in parallel with the 20 $\Omega$. Also transform the $-60$ V source in series with the 5 $\Omega$ resistor into a $-12$ A source in parallel with the 5 $\Omega$ resistor. The result is shown in figure 1.

- Combine the three current sources into a single current source, using KCL, and combine the 20 $\Omega$, 5 $\Omega$, and 6 $\Omega$ resistors in parallel. The result is shown in figure 2.

- To simplify the circuit further, transform the resulting 30 A source in parallel with the 2.4 $\Omega$ resistor into a 72 V source in series with the 2.4 $\Omega$ resistor. Combine the 2.4 $\Omega$ resistor in series with the 1.6 $\Omega$ resistor to get a simple circuit that still maintains the voltage $v$. The result is shown in figure 3.
Continued (Assessment 4.15):

\[ i = \frac{72}{12} = 6 \text{ A} \]
\[ v = 6(8) = 48 \text{ V} \]

b) Use \( i \) to calculate \( v_a \) in the second circuit:

\[ v_a = 6(1.6 + 8) = 57.6 \text{ V} \]

Returning back to the original circuit:

\[ i_a = \frac{120 - v_a}{20} = \frac{120 - 57.6}{20} = 3.12 \text{ A} \]

\[ p_{120\text{ V}} = -(120)i_a = -(120)(3.12) = -374.40 \text{ W} \quad \text{(delivers)} \]
• **Question 4.59:**

  a) Use a series of source transformations to find the current $i_o$ in the circuit in Fig. P4.59.

  b) Verify your solution by using the node-voltage method to find $i_o$.

• **Solution:**

  a) Apply source transformations to both current sources to get:

  $$i_o = \frac{13.8 + 4.2}{2700 + 2300 + 1000} = 3 \text{ mA}$$

  b) The node voltage equations:

  $$6 \times 10^{-3} + \frac{v_1}{2300} + \frac{v_1 - v_2}{2700} = 0$$

  $$\frac{v_2}{1000} + \frac{v_2 - v_1}{2700} - 4.2 \times 10^{-3} = 0$$

  Solving, $v_1 = -6.9 \text{ V}$; $v_2 = 1.2 \text{ V}$

  $$\therefore i_o = \frac{v_2 - v_1}{2700} = 3 \text{ mA}$$
Question 4.60:

a) Find the current in the 10 kΩ resistor in the circuit in Fig. P4.60 by making a succession of appropriate source transformations.

b) Using the result obtained in (a), work back through the circuit to find the power developed by the 100 V source.

Solution:

a) We can simplify the circuit through some consecutive transformation as in the following figures:

\[ i_o = \frac{120}{30,000} = 4 \text{ mA} \]
Continued (Question 4.60):

\[v_a = (15,000)(0.004) = 60 \text{ V}\]
\[i_a = \frac{v_a}{60,000} = 1 \text{ mA}\]
\[i_b = 12 - 1 - 4 = 7 \text{ mA}\]
\[v_b = 60 - (0.007)(4000) = 32 \text{ V}\]
\[i_g = 0.007 - \frac{32}{80,000} = 6.6 \text{ mA}\]
\[p_{100V} = -(100)(6.6 \times 10^{-3}) = -660 \text{ mW}\]
• **Question 4.62:**

  a) Use a series of *source transformations* to find $i_o$ in the circuit in Fig. P4.62.

  b) Verify your solution by using the *mesh-current* method to find $i_o$.

**Solution:**

a) Applying a source transformation to each current source yields:

Now combine the 20 V and 10 V sources into a single voltage source and the 5 $\Omega$, 4 $\Omega$ and 1 $\Omega$ resistors into a single resistor to get:

Now use a source transformation on each voltage source, thus...
Continued (Question 4.62):

Which can be reduced to:

\[
\therefore i_o = \frac{(1.25)(8)}{10} = 1 \text{ A}
\]

b)

\[
5(i_a - 4) + 4i_a + 1(i_a - 10) + 40(i_a - i_b) + 10 = 0
\]

\[
-10 + 40(i_b - i_a) + 2i_b = 0
\]

\[\text{Solving: } i_a = 0.8 \text{ A, } i_b = 1 \text{ A} = i_o\]